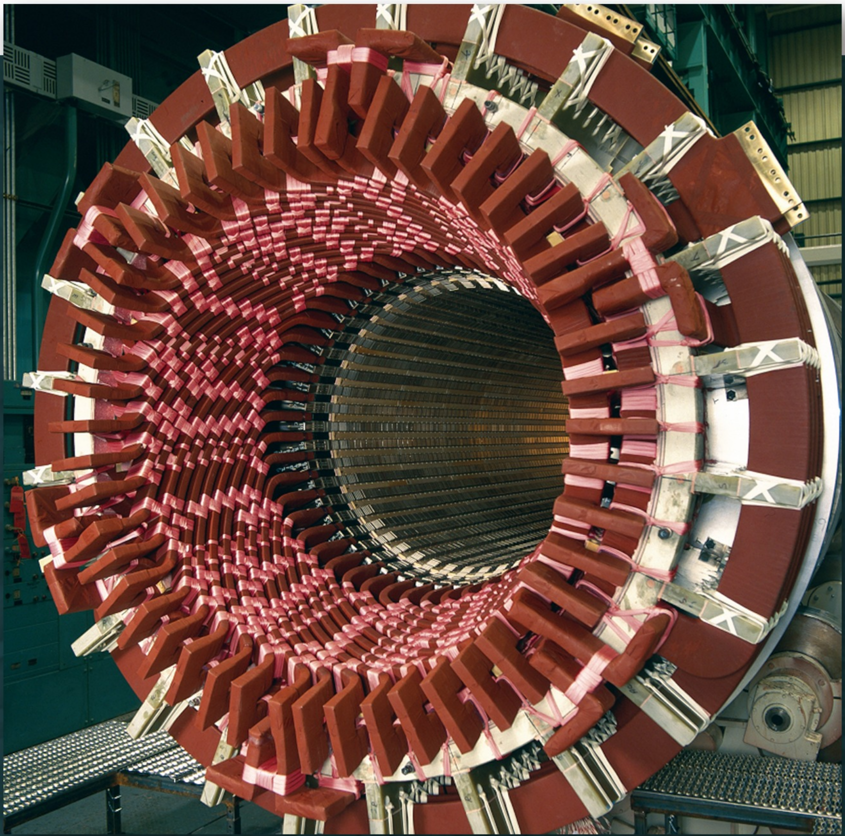
Fitzgerald & Kingsley's

ELECTRIC MACHINERY



Seventh Edition

STEPHEN D. UMANS

FITZGERALD & KINGSLEY'S

Electric Machinery

Seventh Edition

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FITZGERALD & KINGSLEY'S ELECTRIC MACHINERY, SEVENTH EDITION

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This edition of Electric Machinery is dedicated to
Professor Gerald Wilson, my teacher, mentor and dear friend.

ABOUT THE AUTHOR

Stephen D. Umans is a graduate of the Massachusetts Institute of Technology, from which he received the S.B., S.M., E.E., and Sc.D. degrees, all in electrical engineering. He is currently engaged as an independent consultant, having previously held the position of Principal Research Engineer in the MIT Electromechanical Systems Laboratory and lecturer in the MIT Department of Electrical Engineering and Computer Science. He is a member of the US National Academy of Engineering, a Fellow of the IEEE and a recipient of the Cyril Veinott Electromechanical Energy Conversion Award from the IEEE Power Engineering Society.

BRIEF CONTENTS

CONTENTS

PREFACE

Since Professors Fitzgerald & Kingsley first published the first edition in 1952, a consistent theme of ELECTRIC MACHINERY has been an emphasis on the development of both physical insight into the characteristics of electric machinery as well as facility with the analytical techniques used to describe their performance. Much has changed since the publication of the first edition; the development of lower-loss electrical steels, rare-earth permanent-magnet materials, improvements in manufacturing techniques, and the advent of power-electronic control and drive systems. The net result is that modern electric machines achieve higher efficiency and are found in an ever-increasing number of applications.

However, the basic principles which govern the performance of electric machinery remain unchanged. The long-standing reputation of ELECTRIC MACHINERY stems in great part from the emphasis on these fundamentals principles. The challenge in producing each new edition is to appropriately "modernize" the treatment while retaining this basic focus. Modernization in previous editions has included an introduction of rare-earth permanent-magnet materials, the inclusion of permanent-magnet ac machines, variable-reluctance machines, and stepping motors as well as a discussion of field-oriented control algorithms.

A significant addition to the sixth edition was the introduction of MATLAB® for use in examples and practice problems as well as in end-of-chapter problems. MATLAB[1] is widely used in many universities and is available in a student version[2]. Although very little in the way of sophisticated mathematics is required of the reader of ELECTRIC MACHINERY, the mathematics can get somewhat messy and tedious. This is especially true in the analysis of ac machines in which there is a significant amount of algebra involving complex numbers. Analytic tools such as MATLAB can relieve the student of having to perform lengthy calculations which in themselves do little to enhance understanding.

Consider: At the time of the publication of the first edition, the chief computational tool available to students was the slide rule. Using only a slide rule, calculating the performance of an induction motor at a single load point, which involves solving equations with complex arguments, is a significant task which can be quite time consuming and which leaves many opportunities for calculation error.

Fast forward to 2013. A MATLAB script to solve the same problem can be easily written and debugged in a matter of minutes, with the solution then obtained essentially instantaneously. With only a slight modification, the same script can be

[1] MATLAB and Simulink are registered trademarks of The MathWorks, Inc., 3 Apple Hill Drive, Natick, MA 01760 (http://www.mathworks.com).

[2] The MATLAB Student Version is published and distributed by The MathWorks, Inc. (http://www.mathworks.com).

used to calculate, plot and investigate the performance of the motor over its complete operating range as well as to study the effects of parameter changes, etc.; a task which, if performed with a slide rule (or even a calculator), would require the repeated calculation of many operating points, with each calculation as time consuming as the first.

It should be emphasized that although MATLAB has been chosen for ELECTRIC MACHINERY, equivalent alternative numerical-analysis programs, of which there are many, can be used with equal effectiveness. The key point is that the use of such programs immensely reduces the computational burden on the student and thus significantly increases his/her ability to focus on the principles under consideration.

Note that, even in cases where it is not specifically suggested, most of the end-of-chapter problems in the book can be worked using MATLAB or an equivalent program. Thus, students who are comfortable using such tools should be encouraged to do so to save themselves the need to grind through messy calculations by hand. When solving homework problems, students should still of course be required to show on paper how they formulated their solution, since it is the formulation of the solution that is key to understanding the material. However, once a problem is properly formulated, there is typically little additional to be learned from the number crunching itself. The value of working examples and end-of-chapter problems is derived primarily from the process of formulating the solution and from examining the results.

In addition, at the time the revision of the sixth edition was underway, topics related to energy conversion were being re-introduced into a number of engineering curricula. Feedback from faculty involved in these programs led to the inclusion of a chapter which covered the basic principles of power electronics with an emphasis on their application to electric machines. The power-electronics chapter was of course not intended to be a substitute for a full-fledged course in power electronics. At this time, such courses exist in many engineering programs. Faculty members surveyed in advance of this seventh edition indicated that there is no longer a need for the power-electronics chapter. As a result, it has been removed from the seventh edition and moved to the seventh-edition website.

In considering this revision, there was no question of any change in the focus on fundamental physical principles underlying the performance of electric machines which has been the strength of ELECTRIC MACHINERY since the first edition. In addition, a survey of current adopters of the sixth edition indicated that, with the exception of elimination of the chapter on power electronics, there was no need to revise the range of topics covered. On the other hand, elimination of the power-electronics chapter resulted in space for expansion. Thus, the key features of this revision are:

- The presentation of all material in the book has been carefully reviewed, revised and/or expanded as needed for additional clarity. One such example is the expanded treatment of permanent-magnet ac machines in Chapter 5. Similarly, the dc-machine presentation of Chapter 7 has been reorganized for added clarity.

- 15 new examples have been added to this edition, bringing the total number of examples in the book to 111, and in addition, some of the examples from the previous edition have been revised.
- Of the total of 371 end-of-chapter problems in this edition, 96 are new problems. Almost all of the remaining problems, although retained in form from the previous edition, have been altered either in substance or numerically such that previous solutions are no longer valid.
- The use of MATLAB has been considerably expanded in the seventh edition, in examples, in practice problems and in end-of-chapter problems.
- New to this edition is a list of variables and their definitions included at the end of each chapter.
- The seventh edition introduces some simple examples of electric-machinery dynamics and includes a few MATLAB/Simulink® examples and problems.
- The majority of photographs from the previous edition have been updated.

As has been the case with past editions, it is highly likely that there is simply too much material in this edition of ELECTRIC MACHINERY for a single introductory course. The material has been organized so that instructors can pick and choose material appropriate to the topics which they wish to cover. The first two chapters introduce basic concepts of magnetic circuits, magnetic materials and transformers. The third chapter introduces the basic concept of electromechanical energy conversion. The fourth chapter then provides an overview of, and introduction to, the various machine types. Some instructors may choose to omit all or most of the material in Chapter 3 from an introductory course. This can be done without significantly impacting the presentation of the material in the remainder of the book.

The next five chapters provide a more in-depth discussion of the various machine types: synchronous machines in Chapter 5, induction machines in Chapter 6, dc machines in Chapter 7, variable-reluctance machines in Chapter 8 and single/two-phase machines in Chapter 9. Since the chapters are relatively independent (with the exception of the material in Chapter 9 which builds upon the polyphase-induction-motor discussion of Chapter 6), the order of these chapters can be changed and/or an instructor can choose to focus on only one or two machine types and not to cover the material in all five of these chapters.

Finally, instructors may wish to select topics from the control material of Chapter 10 rather than include it all. The material on speed control is a relatively straightforward extension of the material found in earlier chapters on the individual machine types. The material on field-oriented control requires a somewhat more sophisticated understanding and builds upon the dq0 transformation found in Appendix C. It would certainly be reasonable to omit this material in an introductory course and to delay it for a more advanced course where sufficient time is available to devote to it.

I would like to specifically acknowledge Prof. Charles Brice of the University of South Carolina and Prof. Gerald Brown of Cedarville University who carefully reviewed various sections of the draft and caught a number of typos and numerical

errors. I also wish to thank the many other reviewers who provided feedback during the planning process of this revision.

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Visit the textbook website at **www.mhhe.com/umans7e**. The sixth edition Power Electronics chapter has been posted to the website. For instructors, a downloadable version of the solutions manual, PowerPoint slides of figures from the book, and PowerPoint lecture outlines are posted to the Instructor Edition. Copies of the MATLAB and Simulink files for the various examples used in the book are available for students and instructors.

My mother, Nettie Umans, passed away during the time of this revision. I had looked forward to sharing the seventh edition with her; she would have been excited to see it. She is deeply missed.

Stephen D. Umans
Belmont, MA
2013

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CHAPTER 1

Magnetic Circuits and Magnetic Materials

The objective of this book is to study the devices used in the interconversion of electric and mechanical energy. Emphasis is placed on electromagnetic rotating machinery, by means of which the bulk of this energy conversion takes place. However, the techniques developed are generally applicable to a wide range of additional devices including linear machines, actuators, and sensors.

Although not an electromechanical-energy-conversion device, the transformer is an important component of the overall energy-conversion process and is discussed in Chapter 2. As with the majority of electromechanical-energy-conversion devices discussed in this book, magnetically coupled windings are at the heart of transformer performance. Hence, the techniques developed for transformer analysis form the basis for the ensuing discussion of electric machinery.

Practically all transformers and electric machinery use ferro-magnetic material for shaping and directing the magnetic fields which act as the medium for transferring and converting energy. Permanent-magnet materials are also widely used in electric machinery. Without these materials, practical implementations of most familiar electromechanical-energy-conversion devices would not be possible. The ability to analyze and describe systems containing these materials is essential for designing and understanding these devices.

This chapter will develop some basic tools for the analysis of magnetic field systems and will provide a brief introduction to the properties of practical magnetic materials. In Chapter 2, these techniques will be applied to the analysis of transformers. In later chapters they will be used in the analysis of rotating machinery.

In this book it is assumed that the reader has basic knowledge of magnetic and electric field theory such as is found in a basic physics course for engineering students. Some readers may have had a course on electromagnetic field theory based on Maxwell's equations, but an in-depth understanding of Maxwell's equations is not a prerequisite for mastery of the material of this book. The techniques of magnetic-circuit analysis which provide algebraic approximations to exact field-theory solutions are widely used in the study of electromechanical-energy-conversion devices and form the basis for most of the analyses presented here.

1.1 INTRODUCTION TO MAGNETIC CIRCUITS

The complete, detailed solution for magnetic fields in most situations of practical engineering interest involves the solution of Maxwell's equations and requires a set of constitutive relationships to describe material properties. Although in practice exact solutions are often unattainable, various simplifying assumptions permit the attainment of useful engineering solutions.[1]

We begin with the assumption that, for the systems treated in this book, the frequencies and sizes involved are such that the displacement-current term in Maxwell's equations can be neglected. This term accounts for magnetic fields being produced in space by time-varying electric fields and is associated with electromagnetic radiation. Neglecting this term results in the magneto-quasi-static form of the relevant Maxwell's equations which relate magnetic fields to the currents which produce them.

$$\oint_C \mathbf{H}d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{a} \tag{1.1}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{a} = 0 \tag{1.2}$$

Equation 1.1, frequently referred to as *Ampere's Law*, states that the line integral of the tangential component of the *magnetic field intensity* $\mathbf{H}$ around a closed contour C is equal to the total current passing through any surface S linking that contour. From Eq. 1.1 we see that the source of $\mathbf{H}$ is the *current density* $\mathbf{J}$. Eq. 1.2, frequently referred to as *Gauss' Law for magnetic fields*, states that *magnetic flux density* $\mathbf{B}$ is conserved, i.e., that no net flux enters or leaves a closed surface (this is equivalent to saying that there exist no monopolar sources of magnetic fields). From these equations we see that the magnetic field quantities can be determined solely from the instantaneous values of the source currents and hence that time variations of the magnetic fields follow directly from time variations of the sources.

A second simplifying assumption involves the concept of a *magnetic circuit*. It is extremely difficult to obtain the general solution for the magnetic field intensity $\mathbf{H}$ and the magnetic flux density $\mathbf{B}$ in a structure of complex geometry. However, in many practical applications, including the analysis of many types of electric machines, a three-dimensional field problem can often be approximated by what is essentially

[1] Computer-based numerical solutions based upon the finite-element method form the basis for a number of commercial programs and have become indispensable tools for analysis and design. Such tools are typically best used to refine initial analyses based upon analytical techniques such as are found in this book. Because such techniques contribute little to a fundamental understanding of the principles and basic performance of electric machines, they are not discussed in this book.

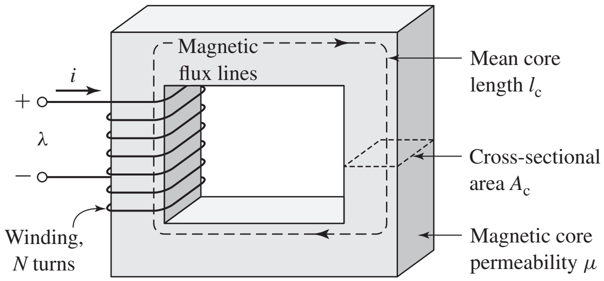

Figure 1.1 Simple magnetic circuit. λ is the winding flux linkage as defined in Section 1.2.

a one-dimensional circuit equivalent, yielding solutions of acceptable engineering accuracy.

A magnetic circuit consists of a structure composed for the most part of high-permeability magnetic material.[2] The presence of high-permeability material tends to cause magnetic flux to be confined to the paths defined by the structure, much as currents are confined to the conductors of an electric circuit. Use of this concept of the magnetic circuit is illustrated in this section and will be seen to apply quite well to many situations in this book.[3]

A simple example of a magnetic circuit is shown in Fig. 1.1. The core is assumed to be composed of magnetic material whose *magnetic permeability* μ is much greater than that of the surrounding air ($\mu \gg \mu_0$) where $\mu_0 = 4\pi \times 10^{-7}$ H/m is the magnetic permeability of free space. The core is of uniform cross section and is excited by a winding of N turns carrying a current of i amperes. This winding produces a magnetic field in the core, as shown in the figure.

Because of the high permeability of the magnetic core, an exact solution would show that the magnetic flux is confined almost entirely to the core, with the field lines following the path defined by the core, and that the flux density is essentially uniform over a cross section because the cross-sectional area is uniform. The magnetic field can be visualized in terms of flux lines which form closed loops interlinked with the winding.

As applied to the magnetic circuit of Fig. 1.1, the source of the magnetic field in the core is the ampere-turn product Ni. In magnetic circuit terminology Ni is the *magnetomotive force* (mmf) $\mathcal{F}$ acting on the magnetic circuit. Although Fig. 1.1 shows only a single winding, transformers and most rotating machines typically have at least two windings, and Ni must be replaced by the algebraic sum of the ampere-turns of all the windings.

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[2] In its simplest definition, magnetic permeability can be thought of as the ratio of the magnitude of the magnetic flux density B to the magnetic field intensity H.

[3] For a more extensive treatment of magnetic circuits see A.E. Fitzgerald, D.E. Higgenbotham, and A. Grabel, *Basic Electrical Engineering*, 5th ed., McGraw-Hill, 1981, chap. 13; also E.E. Staff, M.I.T., *Magnetic Circuits and Transformers*, M.I.T. Press, 1965, chaps. 1 to 3.

The net *magnetic flux* ϕ crossing a surface S is the surface integral of the normal component of **B**; thus

$$\phi = \int_S \mathbf{B} \cdot \mathbf{da} \tag{1.3}$$

In SI units, the unit of ϕ is the *weber* (Wb).

Equation 1.2 states that the net magnetic flux entering or leaving a closed surface (equal to the surface integral of **B** over that closed surface) is zero. This is equivalent to saying that all the flux which enters the surface enclosing a volume must leave that volume over some other portion of that surface because magnetic flux lines form closed loops. Because little flux "leaks" out the sides of the magnetic circuit of Fig. 1.1, this result shows that the net flux is the same through each cross section of the core.

For a magnetic circuit of this type, it is common to assume that the magnetic flux density (and correspondingly the magnetic field intensity) is uniform across the cross section and throughout the core. In this case Eq. 1.3 reduces to the simple scalar equation

$$\phi_c = B_c A_c \tag{1.4}$$

where

$\phi_c =$ core flux
$B_c =$ core flux density
$A_c =$ core cross-sectional area

From Eq. 1.1, the relationship between the mmf acting on a magnetic circuit and the magnetic field intensity in that circuit is.[4]

$$\mathcal{F} = Ni = \oint \mathbf{Hdl} \tag{1.5}$$

The core dimensions are such that the path length of any flux line is close to the mean core length l_c. As a result, the line integral of Eq. 1.5 becomes simply the scalar product $H_c l_c$ of the magnitude of **H** and the mean flux path length l_c. Thus, the relationship between the mmf and the magnetic field intensity can be written in magnetic circuit terminology as

$$\mathcal{F} = Ni = H_c l_c \tag{1.6}$$

where H_c is average magnitude of **H** in the core.

The direction of H_c in the core can be found from the *right-hand rule*, which can be stated in two equivalent ways. (1) Imagine a current-carrying conductor held in the right hand with the thumb pointing in the direction of current flow; the fingers then point in the direction of the magnetic field created by that current. (2) Equivalently, if the coil in Fig. 1.1 is grasped in the right hand (figuratively speaking) with the fingers

[4] In general, the mmf drop across any segment of a magnetic circuit can be calculated as $\int \mathbf{Hdl}$ over that portion of the magnetic circuit.

pointing in the direction of the current, the thumb will point in the direction of the magnetic fields.

The relationship between the magnetic field intensity **H** and the magnetic flux density **B** is a property of the material in which the field exists. It is common to assume a linear relationship; thus

$$\mathbf{B} = \mu \mathbf{H} \tag{1.7}$$

where μ is the material's magnetic permeability. In SI units, **H** is measured in units of *amperes per meter*, **B** is in *webers per square meter*, also known as *teslas* (T), and μ is in *webers per ampere-turn-meter*, or equivalently *henrys per meter*. In SI units the permeability of free space is $\mu_0 = 4\pi \times 10^{-7}$ henrys per meter. The permeability of linear magnetic material can be expressed in terms of its *relative permeability* μ_r, its value relative to that of free space; $\mu = \mu_r\mu_0$. Typical values of μ_r range from 2,000 to 80,000 for materials used in transformers and rotating machines. The characteristics of ferromagnetic materials are described in Sections 1.3 and 1.4. For the present we assume that μ_r is a known constant, although it actually varies appreciably with the magnitude of the magnetic flux density.

Transformers are wound on closed cores like that of Fig. 1.1. However, energy conversion devices which incorporate a moving element must have air gaps in their magnetic circuits. A magnetic circuit with an air gap is shown in Fig. 1.2. When the air-gap length g is much smaller than the dimensions of the adjacent core faces, the core flux ϕ_c will follow the path defined by the core and the air gap and the techniques of magnetic-circuit analysis can be used. If the air-gap length becomes excessively large, the flux will be observed to "leak out" of the sides of the air gap and the techniques of magnetic-circuit analysis will no longer be strictly applicable.

Thus, provided the air-gap length g is sufficiently small, the configuration of Fig. 1.2 can be analyzed as a magnetic circuit with two series components both carrying the same flux ϕ: a magnetic core of permeability μ, cross-sectional area A_c and mean length l_c, and an air gap of permeability μ_0, cross-sectional area A_g and length g. In the core

$$B_c = \frac{\phi}{A_c} \tag{1.8}$$

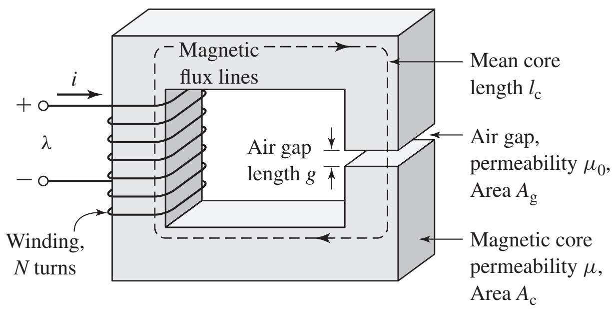

Figure 1.2 Magnetic circuit with air gap.

and in the air gap

$$B_g = \frac{\phi}{A_c} \tag{1.9}$$

Application of Eq. 1.5 to this magnetic circuit yields

$$\mathcal{F} = H_c l_c + H_g g \tag{1.10}$$

and using the linear *B-H* relationship of Eq. 1.7 gives

$$\mathcal{F} = \frac{B_c}{\mu} l_c + \frac{B_g}{\mu_0} g \tag{1.11}$$

Here the $\mathcal{F} = Ni$ is the mmf applied to the magnetic circuit. From Eq. 1.10 we see that a portion of the mmf, $\mathcal{F}_c = H_c l_c$, is required to produce magnetic field in the core while the remainder, $\mathcal{F}_g = H_g g$ produces magnetic field in the air gap.

For practical magnetic materials (as is discussed in Sections 1.3 and 1.4), B_c and H_c are not simply related by a known constant permeability μ as described by Eq. 1.7. In fact, B_c is often a nonlinear, multi-valued function of H_c. Thus, although Eq. 1.10 continues to hold, it does not lead directly to a simple expression relating the mmf and the flux densities, such as that of Eq. 1.11. Instead the specifics of the nonlinear B_c-H_c relation must be used, either graphically or analytically. However, in many cases, the concept of constant material permeability gives results of acceptable engineering accuracy and is frequently used.

From Eqs. 1.8 and 1.9, Eq. 1.11 can be rewritten in terms of the flux ϕ_c as

$$\mathcal{F} = \phi \left(\frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g} \right) \tag{1.12}$$

The terms that multiply the flux in this equation are known as the *reluctance* ($\mathcal{R}$) of the core and air gap, respectively,

$$\mathcal{R}_c = \frac{l_c}{\mu A_c} \tag{1.13}$$

$$\mathcal{R}_g = \frac{g}{\mu_0 A_g} \tag{1.14}$$

and thus

$$\mathcal{F} = \phi(\mathcal{R}_c + \mathcal{R}_g) \tag{1.15}$$

Finally, Eq. 1.15 can be inverted to solve for the flux

$$\phi = \frac{\mathcal{F}}{\mathcal{R}_c + \mathcal{R}_g} \tag{1.16}$$

or

$$\phi = \frac{\mathcal{F}}{\frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g}} \tag{1.17}$$

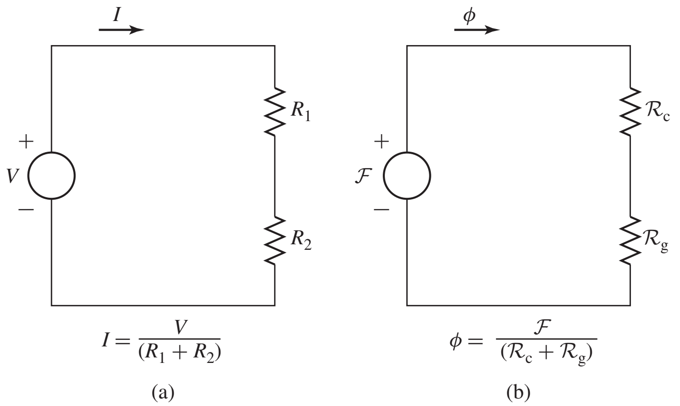

Figure 1.3 Analogy between electric and magnetic circuits. (a) Electric circuit. (b) Magnetic circuit.

In general, for any magnetic circuit of total reluctance $\mathcal{R}_{\text{tot}}$, the flux can be found as

$$\phi = \frac{\mathcal{F}}{\mathcal{R}_{\text{tot}}} \tag{1.18}$$

The term which multiplies the mmf is known as the *permeance* $\mathcal{P}$ and is the inverse of the reluctance; thus, for example, the total permeance of a magnetic circuit is

$$\mathcal{P}_{\text{tot}} = \frac{1}{\mathcal{R}_{\text{tot}}} \tag{1.19}$$

Note that Eqs. 1.15 and 1.16 are analogous to the relationships between the current and voltage in an electric circuit. This analogy is illustrated in Fig. 1.3. Figure 1.3a shows an electric circuit in which a voltage V drives a current I through resistors R_1 and R_2. Figure 1.3b shows the schematic equivalent representation of the magnetic circuit of Fig. 1.2 . Here we see that the mmf $\mathcal{F}$ (analogous to voltage in the electric circuit) drives a flux ϕ (analogous to the current in the electric circuit) through the combination of the reluctances of the core $\mathcal{R}_c$ and the air gap $\mathcal{R}_g$. This analogy between the solution of electric and magnetic circuits can often be exploited to produce simple solutions for the fluxes in magnetic circuits of considerable complexity.

The fraction of the mmf required to drive flux through each portion of the magnetic circuit, commonly referred to as the *mmf drop* across that portion of the magnetic circuit, varies in proportion to its reluctance (directly analogous to the voltage drop across a resistive element in an electric circuit). Consider the magnetic circuit of Fig. 1.2. From Eq. 1.13 we see that high material permeability can result in low core reluctance, which can often be made much smaller than that of the air gap; i.e., for $(\mu A_c/l_c) \gg (\mu_0 A_g/g)$, $\mathcal{R}_c \ll \mathcal{R}_g$ and thus $\mathcal{R}_{\text{tot}} \approx \mathcal{R}_g$. In this case, the reluctance of the core can be neglected and the flux can be found from Eq. 1.16 in terms of $\mathcal{F}$ and the air-gap properties alone:

$$\phi \approx \frac{\mathcal{F}}{\mathcal{R}_g} = \frac{\mathcal{F}\mu_0 A_g}{g} = Ni\,\frac{\mu_0 A_g}{g} \tag{1.20}$$

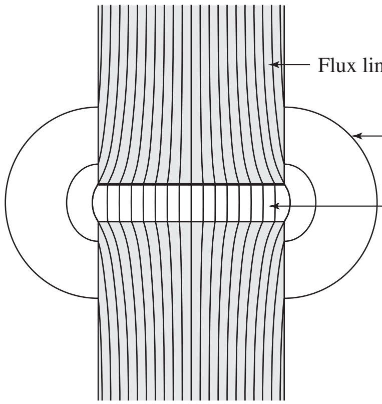

Figure 1.4 Air-gap fringing fields.

As will be seen in Section 1.3, practical magnetic materials have permeabilities which are not constant but vary with the flux level. From Eqs. 1.13 to 1.16 we see that as long as this permeability remains sufficiently large, its variation will not significantly affect the performance of a magnetic circuit in which the dominant reluctance is that of an air gap.

In practical systems, the magnetic field lines "fringe" outward somewhat as they cross the air gap, as illustrated in Fig. 1.4. Provided this fringing effect is not excessive, the magnetic-circuit concept remains applicable. The effect of these *fringing fields* is to increase the effective cross-sectional area A_g of the air gap. Various empirical methods have been developed to account for this effect. A correction for such fringing fields in short air gaps can be made by adding the gap length to each of the two dimensions making up its cross-sectional area. In this book the effect of fringing fields is usually ignored. If fringing is neglected, $A_g = A_c$.

In general, magnetic circuits can consist of multiple elements in series and parallel. To complete the analogy between electric and magnetic circuits, we can generalize Eq. 1.5 as

$$\mathcal{F} = \oint \mathbf{H}d\mathbf{l} = \sum_k \mathcal{F}_k = \sum_k H_k l_k \tag{1.21}$$

where $\mathcal{F}$ is the mmf (total ampere-turns) acting to drive flux through a closed loop of a magnetic circuit, and $\mathcal{F}_k = H_k l_k$ is the *mmf drop* across the k'th element of that loop. This is directly analogous to Kirchoff's voltage law for electric circuits consisting of voltage sources and resistors

$$V = \sum_k R_k i_k \tag{1.22}$$

where V is the source voltage driving current around a loop and $R_k i_k$ is the voltage drop across the k'th resistive element of that loop.

Similarly, the analogy to Kirchoff's current law

$$\sum_n i_n = 0 \tag{1.23}$$

which says that the net current, i.e. the sum of the currents, into a node in an electric circuit equals zero is

$$\sum_n \phi_n = 0 \tag{1.24}$$

which states that the net flux into a node in a magnetic circuit is zero.

We have now described the basic principles for reducing a magneto-quasi-static field problem with simple geometry to a *magnetic circuit model*. Our limited purpose in this section is to introduce some of the concepts and terminology used by engineers in solving practical design problems. We must emphasize that this type of thinking depends quite heavily on engineering judgment and intuition. For example, we have tacitly assumed that the permeability of the "iron" parts of the magnetic circuit is a constant known quantity, although this is not true in general (see Section 1.3), and that the magnetic field is confined solely to the core and its air gaps. Although this is a good assumption in many situations, it is also true that the winding currents produce magnetic fields outside the core. As we shall see, when two or more windings are placed on a magnetic circuit, as happens in the case of both transformers and rotating machines, these fields outside the core, referred to as *leakage fields*, cannot be ignored and may significantly affect the performance of the device.

EXAMPLE 1.1

The magnetic circuit shown in Fig. 1.2 has dimensions $A_c = A_g = 9\ \text{cm}^2$, $g = 0.050$ cm, $l_c = 30$ cm, and $N = 500$ turns. Assume the value $\mu_r = 70{,}000$ for core material. (a) Find the reluctances $\mathcal{R}_c$ and $\mathcal{R}_g$. For the condition that the magnetic circuit is operating with $B_c = 1.0$ T, find (b) the flux ϕ and (c) the current i.

■ Solution

a. The reluctances can be found from Eqs. 1.13 and 1.14:

$$\mathcal{R}_c = \frac{l_c}{\mu_r \mu_0 A_c} = \frac{0.3}{70{,}000\ (4\pi \times 10^{-7})(9 \times 10^{-4})} = 3.79 \times 10^3 \ \frac{\text{A}\cdot\text{turns}}{\text{Wb}}$$

$$\mathcal{R}_g = \frac{g}{\mu_0 A_g} = \frac{5 \times 10^{-4}}{(4\pi \times 10^{-7})(9 \times 10^{-4})} = 4.42 \times 10^5 \ \frac{\text{A}\cdot\text{turns}}{\text{Wb}}$$

b. From Eq. 1.4,

$$\phi = B_c A_c = 1.0(9 \times 10^{-4}) = 9 \times 10^{-4}\ \text{Wb}$$

c. From Eqs. 1.6 and 1.15,

$$i = \frac{\mathcal{F}}{N} = \frac{\phi(\mathcal{R}_c + \mathcal{R}_g)}{N} = \frac{9 \times 10^{-4}(4.46 \times 10^5)}{500} = 0.80\ \text{A}$$